Case A: Both W/PH and W/PL are on the upperward sloped section of the supply of labor,

$$E\Pi(W) := (X)\cdot\left[\left(\frac{W}{PL}\right)^{(\theta\cdot\psi)} - \frac{W}{PL}\cdot\left(\frac{W}{PL}\right)^{\psi}\right] + (1-X)\cdot\left[\left(\frac{W}{PH(X)}\right)^{(\theta\cdot\psi)} - \frac{W}{PH(X)}\cdot\left(\frac{W}{PH(\right.$$

$$\frac{d}{dW}E\Pi(W) \rightarrow X\cdot\left[\left(\frac{W}{PL}\right)^{\theta\cdot\psi}\cdot\theta\cdot\frac{\psi}{W} - \frac{1}{PL}\cdot\left(\frac{W}{PL}\right)^{\psi} - \frac{1}{PL}\cdot\left(\frac{W}{PL}\right)^{\psi}\cdot\psi\right] + (1-X)\cdot\left[\left(\frac{W}{PH(\right.\right.$$

$$0 = \frac{d}{dW}E\Pi(W) \text{ solve}, W \rightarrow \exp\left[\frac{\ln\left[-(1+\psi)\cdot\frac{-X\cdot PH(X)\cdot\exp\left(\psi\cdot\ln\left(\frac{PH(X)}{PL}\right)\right) - F}{\left(X\cdot\exp\left(\theta\cdot\psi\cdot\ln\left(\frac{PH(X)}{PL}\right)\right)+1-X\right)\cdot}\right.}{-1-\psi}\right.$$

As a benchmark consider the deterministic solution when X=0 both symbolically and numerically below for the monopsonist and the competitive firm. For the parameter values assumed below the value of wc is 0.792

For the monopsonist the expected profit is;

$$E\Pi(X) := \left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\theta\cdot\psi} - \frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\cdot\left[\frac{}{\left[(1-\right.}\right.$$

$$\frac{d}{dW}E\Pi(X) \rightarrow \left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\right]^{\theta\cdot\psi}\cdot\theta\cdot\psi\cdot\frac{\frac{1}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}} + \frac{}{\left[(1-k)\cdot\right.}}{W}$$

$$0 = \frac{d}{dW} E\Pi(X) \text{ solve}, W \rightarrow \exp\left(\frac{-\ln\left(-MH(X)\cdot\frac{-1+k}{1+\psi}\cdot\theta\cdot\psi\right) - \ln\left(-MH(X)\cdot\frac{-1+k}{1+\psi}\cdot\theta \right.}{-1 -} \right.$$

This would not converge to a symbolic (or numerical)solution, when I used the parameter values below and this method to find the opt. W. This is because the exp.profit fn. is too flat and many W are the same. I use a different method based upon the r.h.s. equaling the opt.W.

$\theta := .6 \quad \psi := .9 \quad MH(X) := 3.685 \quad k := .5$

$$\exp\left(\frac{-\ln\left(-MH(0)\cdot\frac{-1+k}{1+\psi}\cdot\theta\cdot\psi\right) - \ln\left(-MH(0)\cdot\frac{-1+k}{1+\psi}\cdot\theta\cdot\psi\right)\cdot\psi + \ln\left(-MH(0)\cdot\frac{-1+k}{1+\psi}\cdot\theta\cdot\right.}{-1-\psi+\theta\cdot\psi} \right.$$

Now solve for the profit for the det. case for a firm that sets the competitive wage.wc.

$X := .0 \quad \psi := .9 \quad wc := 0.792 \quad \theta := .6 \quad E\Pi(wc) := \left[(wc)^{(\theta\cdot\psi)} - wc\cdot(wc)^{\psi} \right]$

$E\Pi(wc) = 0.240$ This is the value of exp.profit for a firm for the comp. wage.

Now solve (19) and (20) for PL in terms of ML and W and PH in terms of MH and W, and substute into the opt. W equ. above.First for X = 0, The determistic case for the general specification.

$X := 0$

$Y := (1-k)\cdot\frac{M}{P} \qquad Y := \left(\frac{W}{P}\right)^{(\theta\cdot\psi)} \qquad P := \left[(1-k)\cdot M\cdot W^{-(\theta\cdot\psi)} \right]^{\frac{1}{(1-\theta\cdot\psi)}}$

$k := .5 \quad \theta := .6 \quad \psi := .9 \quad Lo := .5$

$ML := 3.5 \quad Me := 3.685 \quad MH(X) := \frac{(Me - X\cdot ML)}{(1-X)}$

$MH(X) = 3.685$ This makes sense.

$$f(W) := W - \exp\left[\dfrac{\ln\left[-(1+\psi)\cdot\dfrac{-X\cdot\left[(1-k)\cdot\dfrac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}\cdot\exp\left[\psi\cdot\ln\left[\dfrac{\left[(1-k)\cdot\cdots\right.}{\left[(1-k)\cdot\cdots\right.}\right.\right.}{X\cdot\exp\left[\theta\cdot\psi\cdot\ln\left[\dfrac{\left[(1-k)\cdot\dfrac{MH(\cdots}{W^{(\theta\cdot\cdots}}\right.}{\left[(1-k)\cdot\dfrac{ML\cdots}{W^{(\theta\cdot\cdots}}\right.}\right.\right.}\right.}{\cdots}\right.$$

$W := 1$

$W := \mathrm{root}(f(W), W)$ $\qquad W = 1.204$ This is the opt. wage where PH and PL are eliminated in eq. (16) and it is the same as I obtained before.

$$PH(X) := \left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}} \qquad PL := \left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}$$

$PH(X) = 3.036$ $\qquad PL = 2.715$ $\qquad Pe := X\cdot PL + (1-X)\cdot PH(X)$

$Pe = 3.036$ This makes sense.

$$E\Pi(W) := (X)\cdot\left[\left[\frac{W}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{(\theta\cdot\psi)} - \frac{W}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\cdot\left[-\cdots\right.\right.$$

$$\mathrm{root}\left(\frac{d}{dW}E\Pi(W), W\right) = 1.204$$

I have calculated the solution 2 ways and they check. Let us see how flat the exp. profit fn is.

$$W := 1.204$$

Now for the other variables of interest.

$$wc := (Lo)^{(\theta-1)} \cdot (\theta)$$

$$\frac{W}{Pe} = 0.397 \qquad wc = 0.792 \qquad wk := (Lo)^{\frac{1}{\psi}} \qquad \frac{W}{PH(X)} = 0.397$$

$$wk = 0.463 \qquad \frac{W}{PL} = 0.444$$

So we have verified that both W/P are on the supply of labor.

$$LL := \left(\frac{W}{PL}\right)^{\psi}$$

$$LH(X) := \left(\frac{W}{PH(X)}\right)^{\psi} \qquad LH(X) = 0.435$$

$$LL = 0.481$$

$$LH(X) := \left[(1-k)\cdot\frac{MH(X)}{PH(X)}\right]^{\frac{1}{\theta}} \qquad LH(X) = 0.435 \qquad LL^{\theta} = 0.645$$

$$YH(X) := (1-k)\cdot\frac{MH(X)}{PH(X)} \qquad YL := (1-k)\cdot\frac{ML}{PL} \qquad LH(X)^{\theta} = 0.607$$

$$YH(X) = 0.607 \qquad YL = 0.645$$

Now solve for the stocastic case.

I get exactly the same results for X=0.001 as for X=0.

$$X := .2 \qquad Lo := .5$$

$$Y := (1-k)\cdot\frac{M}{P} \qquad Y := \left(\frac{W}{P}\right)^{(\theta\cdot\psi)}$$

$$k := .5 \qquad \theta := .6 \qquad \psi := .9$$

$$P := \left[(1-k)\cdot M\cdot W^{-(\theta\cdot\psi)}\right]^{\frac{1}{(1-\theta\cdot\psi)}}$$

$$ML := 3.5 \qquad Me := 3.685$$

$$MH(X) := \frac{(Me - X\cdot ML)}{(1-X)}$$

$$MH(X) = 3.731$$

$$f(W) := W - \exp\left[\frac{\ln\left[-(1+\psi)\cdot\frac{-X\cdot\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}\cdot\exp\left[\psi\cdot\ln\left[\frac{\left[(1 \cdots\right.}{\left[(\cdots\right.}\right.\right.}{X\cdot\exp\left[\theta\cdot\psi\cdot\ln\left[\frac{\left[(1-k)\cdot \cdots\right.}{\left[(1-k)\cdot \cdots\right.}\right.\right.}\right.}{\cdots}\right.$$

$W := 1$

$W := \mathrm{root}(f(W), W)$ $\quad W = 1.201$ This is the opt. wage.

$$PH(X) := \left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}} \qquad PL := \left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}$$

$PH(X) = 3.128$ $\quad PL = 2.722$ $\quad Pe := X\cdot PL + (1-X)\cdot PH(X)$

$Pe = 3.047$

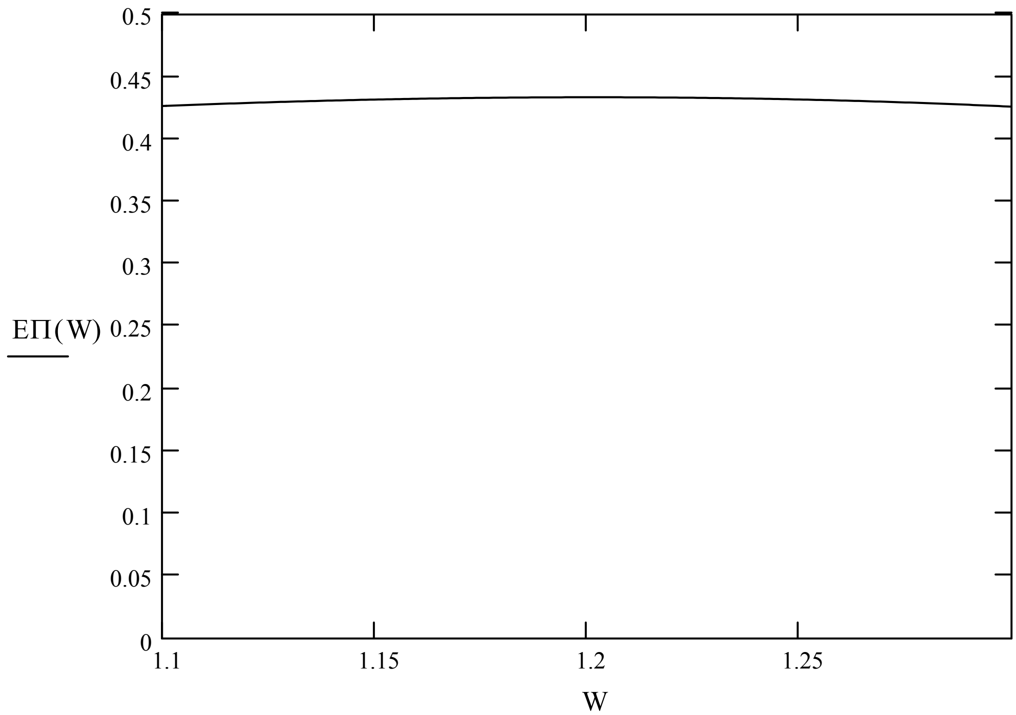
$$E\Pi(W) := (X)\cdot\left[\left[\frac{W}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{(\theta\cdot\psi)} - \frac{W}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot \cdots}}}\right.$$

$$\text{root}\left(\frac{d}{dW}E\Pi(W), W\right) = 1.201$$

I have calculated the solution 2 ways and they check. Let us see how flat the exp. profit fn is.

$W := 1.1, 1.101 .. 1.3$

Let'e see how flat the exp.profit fn. looks.It is VERY flat.



$E\Pi(1.201) = 0.434$

$W := 1.201$

Now for the other variables of interest.

$wc := (Lo)^{(\theta-1)} \cdot (\theta)$ $wc = 0.792$

$wk := (Lo)^{\frac{1}{\psi}}$

$\frac{W}{PH(X)} = 0.384$

$\frac{W}{Pe} = 0.394$ $wk = 0.463$ $\frac{W}{PL} = 0.441$

So we have verified that both W/P are on the supply of labor.

$$LL := \left(\frac{W}{PL}\right)^{\psi}$$

$$LH(X) := \left(\frac{W}{PH(X)}\right)^{\psi} \qquad LH(X) = 0.423$$

$$LL = 0.479$$

$$LH(X) := \left[(1-k)\cdot\frac{MH(X)}{PH(X)}\right]^{\frac{1}{\theta}} \qquad LH(X) = 0.423 \qquad LL^{\theta} = 0.643$$

$$YH(X) := (1-k)\cdot\frac{MH(X)}{PH(X)} \qquad YL := (1-k)\cdot\frac{ML}{PL} \qquad LH(X)^{\theta} = 0.596$$

$$YH(X) = 0.596 \qquad YL = 0.643$$

$$\cdots\left.\frac{\cdots}{\cdots X)}\right)^{\psi}\Bigg]$$

$$\cdots\left.\frac{\cdots}{\cdots X)}\right)^{\theta\cdot\psi}\cdot\theta\cdot\frac{\psi}{W} - \frac{1}{PH(X)}\cdot\left(\frac{W}{PH(X)}\right)^{\psi} - \frac{1}{PH(X)}\cdot\left(\frac{W}{PH(X)}\right)^{\psi}\cdot\psi\Bigg]$$

$$\left.\frac{\cdots\left.\frac{\cdots L + PL\cdot X}{\cdots\psi\cdot\theta\cdot PH(X)}\right] - \psi\cdot\ln\left(\frac{PH(X)}{PL}\right) + \theta\cdot\psi\cdot\ln\left(\frac{PH(X)}{PL}\right)}{\cdots + \theta\cdot\psi}\right]\cdot PL$$

$$\left.\frac{W}{\cdots k)\cdot\left.\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\psi}$$

$$\frac{\frac{1}{\cdots\left.\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}\cdot(1-\theta\cdot\psi)}\cdot\theta\cdot\psi}{\cdots}\cdot\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}} - \frac{1}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\cdot\left[\frac{\cdots}{\left[(\cdots\right.}\right.$$

$$\frac{\cdot\psi\Big)\cdot\psi + \ln\left(-MH(X)\cdot\frac{-1+k}{1+\psi}\cdot\theta\cdot\psi\right)\cdot\theta\cdot\psi - \ln\left(\frac{1+\psi}{\theta\cdot\psi}\right)\cdot\psi}{\psi + \theta\cdot\psi}\Bigg)$$

$$\frac{\psi\Big)\cdot\theta\cdot\psi - \ln\left(\frac{1+\psi}{\theta\cdot\psi}\right)\cdot\psi}{}\Bigg) = 1.204$$

This is the opt. wage for this case.I will use a diff.method to check this result after calculating exp.profit for the Com wage when X=0.

$$\ldots \frac{\ldots\frac{MH(X)}{W^{(\theta\cdot\psi)}}\Big]^{\frac{1}{(1-\theta\cdot\psi)}}}{\ldots\frac{ML}{W^{(\theta\cdot\psi)}}\Big]^{\frac{1}{(1-\theta\cdot\psi)}}}\Bigg]\Bigg] - \Bigg[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}} + \Bigg[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}\cdot X \ldots$$

$$\ldots \frac{\ldots X)}{\ldots\psi)}\Big]^{\frac{1}{(1-\theta\cdot\psi)}} \Big/ \ldots\psi)\Big]^{\frac{1}{(1-\theta\cdot\psi)}}\Bigg]\Bigg] + (1-X)\cdot\Bigg[\psi\cdot\theta\cdot\Bigg[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}\Bigg]\Bigg] - \psi\cdot \ldots$$

$$\ldots \Big/ \left(-1-\psi+\theta\cdot\psi\right)$$

$$\ldots \frac{W}{\Bigg[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}\Bigg]^{\psi}\Bigg] + (1-X)\cdot\Bigg[\Bigg[\frac{W}{\Bigg[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}\Bigg]^{\theta\cdot\psi} - \frac{\ldots}{\Bigg[(1-k)\cdot\ldots} \ldots$$

$$\frac{\dfrac{\cdots 1-k)\cdot\dfrac{MH(X)}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}{\cdots 1-k)\cdot\dfrac{ML}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}\Bigg]\Bigg] - \left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}} + \left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}\cdot X}{\dfrac{\cdots\dfrac{MH(X)}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}{\cdots\dfrac{ML}{W^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}}\Bigg]\Bigg] + (1-X)\Bigg]\cdot\left[\psi\cdot\theta\cdot\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}\right]} \Bigg]$$

$$-1-\psi+\theta\cdot\psi$$

$$\cdots\psi)\;\text{—}\cdot\left[\frac{W}{\left[(1-k)\cdot\dfrac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\psi}\Bigg] + (1-X)\cdot\Bigg[\left[\frac{W}{\left[(1-k)\cdot\dfrac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\theta\cdot\psi} - \frac{\cdots}{\left[(1\cdots\right.}$$

$$\left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\right]^{\psi}-\frac{1}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\cdot\frac{\left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\right]^{\psi}}{1-\theta\cdot\psi}\cdot\theta\cdot\psi\ -$$

$$\ln\left[\frac{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]+\theta\cdot\psi\cdot\ln\left[\frac{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]\Bigg]\cdot\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right.$$

$$\frac{W}{\left[\ldots\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\cdot\left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\psi}\Bigg]$$

$$- \psi \cdot \ln\left[\frac{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right] + \theta\cdot\psi\cdot\ln\left[\frac{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}{\left[(1-k)\cdot\frac{ML}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]\Bigg]$$

$$\overline{}\Bigg]\cdot\Big[\,(1-k)\cdot -$$

$$\frac{W}{- k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\Big]^{\frac{1}{(1-\theta\cdot\psi)}}}\cdot\left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{(\theta\cdot\psi)}}\right]^{\frac{1}{(1-\theta\cdot\psi)}}}\right]^{\psi}\Bigg]$$

$$-\left[\frac{W}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}\right]^{\psi}\cdot\psi\cdot\left[\frac{1}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}}+\frac{1}{\left[(1-k)\cdot\frac{MH(X)}{W^{\theta\cdot\psi}}\right]^{\frac{1}{1-\theta\cdot\psi}}\cdot(}\right.$$

$$\Big]^{\frac{1}{(1-\theta\cdot\psi)}}$$

$$\frac{ML}{N^{(\theta\cdot\psi)}}\Bigg]^{\frac{1}{(1-\theta\cdot\psi)}}$$

$$\overline{} \cdot \theta \cdot \psi \Bigg]$$

$$1 - \theta \cdot \psi) \Bigg]$$