A Reader's Guide to A Stochastic Monopsony Theory of the Business Cycle

Page 12.

Equation (11) is derived from (9) and (10); $dQ_t^j / dL_t^j = d[a_t^j((w_t / w_t^*)^{\phi} \cdot L_t^j)^{\theta}] / dL_t^j =$

$\theta(w_t / w_t^*)^{\phi\theta} \cdot (L_t^j)^{\theta-1} = w_t$, or $[\theta^{-1}(w_t)^{(1-\phi\theta)}(w_t^*)^{\phi\theta}]^{1/(\theta-1)} = L_t^j$.

Page 18.

Equation (14), the expected profit function, upon which the optimal wage decision is based, can be usefully characterized by using heavy side variables. Define $H_D = 1$ if $w_t^c \le w_t$ and the demand for labor is binding. Otherwise the supply of labor is relevant and $H_D = 0$. Define $H_L = 1$, if $w_t^k \le w_t$ and the supply of labor equals N_t^j. Otherwise $H_L = 0$ and $S(w) = w^{\psi}$. Define $H_e = 1$, if $w_t^* \le w_t$ and there is no shirking, $e(w_t) = 1$. Otherwise $H_e = 0$ and $e(w_t) < 1$. Using the heavy side variables and dropping the superscript *j* where obvious to simplify notation, expected profit can be rewritten as

$$E[\Pi] = E[H_D(H_e[(w_t\theta)^{-\theta/(1-\theta)} - ((w_t)^{\theta} / \theta)^{-1/(1-\theta)}] + (1-H_e)\{[(w_t)^{(1-\phi)}(w_t^*)^{\phi\theta} / \theta]^{-\theta/(1-\theta)}$$

$$- [(w_t)^{\theta(1-\phi)}(w_t^*)^{\phi\theta} / \theta]^{-1/(1-\theta)}\}) + (1-H_D)[(H_e + (1-H_e)[(w_t)/(w_t^*)^{\phi}]^{\theta}) \cdot \{(H_L N_t^j + (1-H_L)(w_t)^{\Psi})^{\theta}$$

$$- (w_t)(H_L N_t^j + (1-H_L)(w_t)^{\Psi})\}].$$ or

$$E[\Pi] = H_D[H_e\{X[(W_t / P_t^L\theta)^{-\theta/(1-\theta)} - ((W_t / P_t^L)^{\theta} / \theta)^{-1/(1-\theta)}] + (1-X)[(W_t / P_t^H\theta)^{-\theta/(1-\theta)} -$$

$$((W_t / P_t^H)^{\theta} / \theta)^{-1/(1-\theta)}]\} + (1-H_e)[X[(W_t / P_t^L)^{(1-\phi)}(w_t^*)^{\phi\theta} / \theta]^{-\theta/(1-\theta)} - ((W_t / P_t^L)^{\theta(1-\phi)}(w_t^*)^{\phi\theta} / \theta)^{-1/(1-\theta)})\}]$$

$$+(1-X)\{[(W_t / P_t^H)^{(1-\phi)}(w_t^*)^{\phi\theta} / \theta)]^{-\theta/(1-\theta)} - ((W_t / P_t^H)^{\theta(1-\phi)}(w_t^*)^{\phi\theta} / \theta)^{-1/(1-\theta)}\}] + (1-H_D)[(H_e +$$

$$(1-H_e)\{X((W_t / P_t^L)/(w_t^*)^{\phi})^{\theta} + (1-X)((W_t / P_t^H)/(w_t^*)^{\phi})^{\theta}\} \cdot (H_L N_t^j + (1-H_L)\{X[(W_t / P_t^L)^{\Psi})^{\theta} +$$

$$(1-X)((W_t / P_t^H)^{\Psi})^{\theta}\} - \{X(W_t / P_t^L)(H_L N_t^j + (1-H_L)[W_t / P_t^L]^{\Psi}) + (1-X)(W_t / P_t^H)(H_L N_t^j +$$

$$(1-H_L)[W_t / P_t^H]^{\Psi})\}].$$

The solutions for P_t^H and P_t^L in terms of W_t (and M_t^H or M_t^L) are obtained by equating aggregate demand and aggregate supply. ($L_t^{L,i}$ or $L_t^{H,i}$ is eliminated using equation (7).)

The solution for P_t^H is $P_t^H = [(1-\beta)\cdot M_t^H \cdot (W_t)^{-\theta\psi}]^{1/(1-\theta\psi)}$.

The solution for P_t^L depends upon where employment is determined.

a. The positively sloped section of labor supply: $P_t^L = [(1-\beta)\cdot M_t^L \cdot (W_t)^{-\theta\psi}]^{1/(1-\theta\psi)}$.

b. The vertical section of labor supply: $P_t^L = (1-\beta)\cdot M_t^L \cdot N_t^j$.

c. The demand for labor: $P_t^L = [(1-\beta)\cdot M_t^L]^{(1-\theta)} \cdot (W_t/\theta)^{\theta}$.

Substitution of these functions into equation (16) results in an implicit equation in terms of W_t (and M_t^L or M_t^H). Knowing W_t, the equilibrium values of w_t and P_t^L (P_t^H) are simultaneously determined, in terms of the realizations of M_t^L (M_t^H). Knowing P_t^L (P_t^H), w_t and, therefore, the amount of effort its employees will exert from its knowledge of w_t, the firm determines L_t and Y_t associated with high and low quantities of money.

This model is homogeneous of degree one in nominal values in the quantities of money. Hence, we chose an arbitrary expected value of money, $M^e = 3.685$. The value chosen for M_t^L determines $M_t^H = (M_t^e - \mathrm{X}M_t^L)/(1-\mathrm{X})$. Using parameter values $(\theta, \psi, \phi, w_t^*, k, N_t^j, a_t^j, K)$ = (0.6, 0.9, 0.0, 1, 0.5, 0.5, 1, 1) and X = 0.20, we solve for general equilibrium values of prices, real wages and employment associated with three alternative values of M_t^L. Employment and effort determine output.

a. $M_t^L = 3.5$ and $M_t^H = 3.731$. The optimal nominal wage is $W_t = 1.201$. If M_t^H is realized, $P_t^H = 3.128$, $W_t/P_t^H = 0.384$, and $L_t^H = 0.423$. If M_t^L is realized, $P_t^L = 2.722$, W_t/P_t^L

= 0.441, and $L_t^L = 0.479$. $P_t^e = 3.047$. Employment is determined along the positively sloped section of labor supply, and $L_t^L < N_t^j = 0.5$ but $U_t = 0$.

b. $M_t^L = 3.415$ and $M_t^H = 3.752$. $W_t = 1.198$. If M_t^H is realized, $P_t^H = 3.177$, W_t / P_t^H = 0.377 and $L_t^H = 0.416$. If M_t^L is realized, $P_t^L = 2.588$, $W_t / P_t^L = 0.463 = w_t^k$ (the real wage at which the labor supply curve becomes vertical) and $L_t^L = 0.500 = N_t^j$. $P_t^e = 3.059$. It should be clear from Figure 1 that decreases in the quantity of money M_t^L below 3.41 will decrease P_t^L and increase W_t / P_t^L but do not change employment or output until the transition point $w_t^C = 0.792$ at which point the régime changes from the supply to the demand for labor.

c. $M_t^L = 1.5$ and $M_t^H = 3.752$. $W_t = 1.034$. If M_t^H is realized, $P_t^H = 4.905$, W_t / P_t^H = 0.211, and $L_t^H = 0.246$. If M_t^L is realized, $P_t^H = 4.905$, $W_t / P_t^L = 0.837 > w_t^C = 0.792$ and the labor market is in disequilibrium with employment, $L_t^L = 0.435$, determined by the demand for labor. $L_t^L < N_t^j = 0.5$ and $U_t = 0.65$. $P_t^e = 4.171$. $E[\Pi(W_t)] = 0.434$ exceeds $E[\Pi(w_t^C)] = 0.240$.

Page 19.

Equation (18). To ensure unique solutions, the sign of the Jacobian must be strictly of one sign. This can be justified by any of three arguments.

1. If one imposes the conventional stability restriction that the price should decrease when it is initially higher than equilibrium and there exists an excess supply of goods, and visa versa, then the slope of the aggregate supply must be greater than the slope of aggregate demand and Δ^i must be positive.

2. With continuous functions within a model, if it is recognized that the sign of the Jacobians, Δ^D and Δ^S, should both either be strictly positive or strictly negative, and if the conventional model with a vertical labor supply and no efficiency wage behavior is taken as a paradigm, then $\Delta^D > 0$. By extension, both Δ^D and $\Delta^S > 0$.

3. If one assumes $\Delta^i \neq 0$, then

$$dP_t / dM_t = \left(\Delta^i\right)^{-1} / (1-\beta) P_t .$$

In view of the fact that P_t is a continuous function of M_t, if we are unwilling to exclude the result that increases in the quantity of money cause increases in the price level, $dP_t / dM_t > 0$, it follows that both Δ^D and Δ^S must be positive or the slope of the aggregate supply is greater than that for aggregate demand, even when the aggregate supply is negative. We adopt this assumption. In several cases, this requires that restrictions be placed on the upper bound of the possible parameter values of ϕ^* and ψ^*.

After equation (20), if $H_e = 0$ (as $w_t^* > w_t$), then

$a_{12} = -[K\theta^{1/(1-\theta)}(1-\phi\theta)/(1-\theta)][(w_t)^{(1-\phi\theta)}(w_t^*)^{\phi\theta}]^{-\theta/(1-\theta)}(P_t)^{-1} \leq 0$, if $\phi \leq 1$. More generally in this case, $\Delta^D > 0$, if $\phi < \phi^*$ where

$$\phi \leq \phi^* < (1-\beta)^{-1}[1-\beta+(M_t / P_t)(w_t^*)^{\phi\theta^2/(1-\theta)}\ \theta^{-1/(1-\theta)}(1-\theta)\cdot(w_t)^{\theta(1-\phi\theta)/(1-\theta)} K^{-1}]\theta^{-1}.$$

We use MathCad to solve for equilibrium in the three cases.

Introduction to MATHCAD Files

The MATHCAD files that are included in this folder provide a description of how one solves this model for the (dis) equilibrium price levels, PH and PL, and the optimal wage, W, for a choice of the Bernoulli distribution of, MH, ML, and X. This only deals with the method when efficiency wages are inoperative because it is simpler. The extension to efficiency wage behavior is straightforward, if tedious.

The analysis begins in the neighborhood of the deterministic solution, which is not a competitive equilibrium, but rather lies on the upward-sloped section of the supply of labor below the competitive equilibrium. It is important to recognize that all the relevant functions are monotonic in the variables of interest.

When X= 0 approximately, the problem is considerably more complex because equations (19) and (20) for aggregate supply and aggregate demand must be solved simultaneously with the optimal choice of the nominal wage by all firms (equation 16). Equations (19) and (20) and the production function (equation 11), are used to eliminate Y and L in terms of MH, ML, X, W, PH, and PL. One must recognize that MH>Me>ML and PH>Pe>PL and begin near X= 0. In this event, MH will generate a W/PH that is close to the deterministic solution for firms and intersect the upward sloped section of the labor supply curve.

Three cases are analyzed.

Case A: Both W/PH and W/PL are on the upward-sloped section of the supply of labor.
Case B: W/PH is on this part of the supply and W/PL is on the vertical portion.
Case C: W/PH is on this part of the supply and W/PL is on the demand for labor.

ɔat Docuɔat Docuɔat Docu