Useful Algebraic Facts

Laws of Exponents

$$a^x \cdot a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$
$$a^x \cdot b^x = (ab)^x$$

Quadratic Formula

The solutions of the quadratic equation

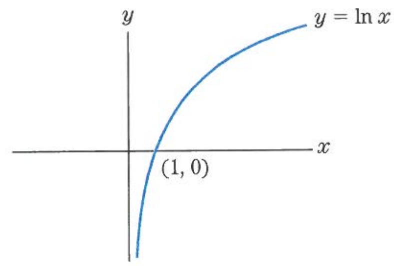
$$ax^2 + bx + c = 0, \quad a \neq 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Logarithms

$$\ln 1 = 0$$
$$\ln e = 1$$
$$\ln x^a = a \ln x$$
$$\ln xy = \ln x + \ln y$$



Factorization and Product Formulas

FOIL:

$$(a + b)(c + d) = \underbrace{ac}_{\text{First}} + \underbrace{ad}_{\text{Outside}} + \underbrace{bc}_{\text{Inside}} + \underbrace{bd}_{\text{Last}}$$

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

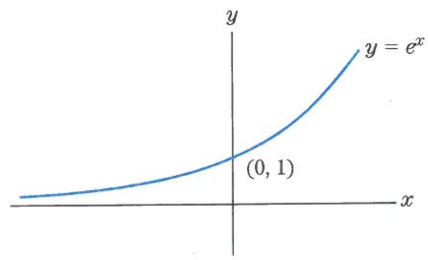
Rational Expressions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$
$$\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

The Natural Exponential Function

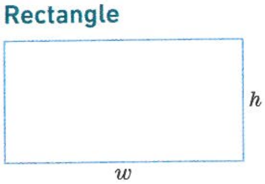
$e = 2.718281\ldots$

Nature exponential function $= e^x$.



Useful Geometric Formulas

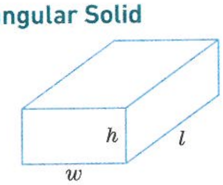
Rectangle



Perimeter $= 2w + 2h$

Area $= wh$

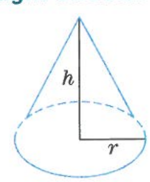
Rectangular Solid



Volume $= lwh$

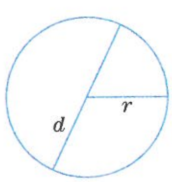
Surface area $= 2wh + 2wl + 2lh$

Right Circular Cone



Volume $= \frac{1}{3}\pi r^2 h$

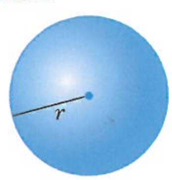
Circle



Circumference $= 2\pi r = \pi d$

Area $= \pi r^2 = \frac{1}{4}\pi d^2$

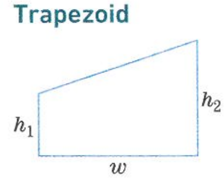
Sphere



Volume $= \frac{4}{3}\pi r^3$

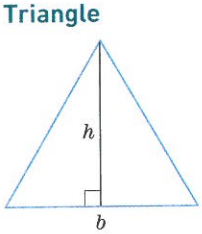
Surface area $= 4\pi r^2$

Trapezoid



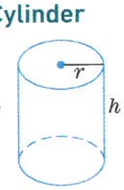
Area $= \dfrac{(h_1 + h_2)}{2} w$

Triangle



Area $= \frac{1}{2}bh$

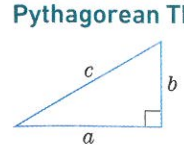
Right Cylinder



Volume $= \pi r^2 h$

Surface area (no top or bottom) $= 2\pi rh$

Pythagorean Theorem



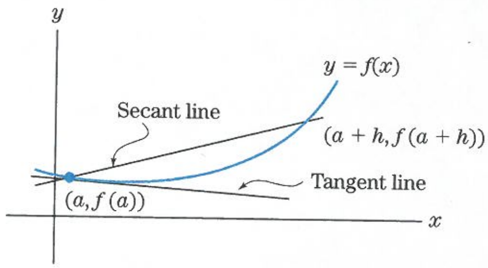
$$a^2 + b^2 = c^2$$

Useful Differentiation Facts

Definition of Derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Secant-Line Approximation of Derivative



$f'(a) =$ slope of tangent line at $(a, f(a))$
As $h \to 0$, slope of secant line approaches slope of tangent line.

Rules for Differentiation

Sum Rule: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Constant Multiple Rule: $\frac{d}{dx}[kf(x)] = k \cdot \frac{d}{dx}[f(x)]$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $y = f(u)$ with $u = g(x)$.

The Differential

One Variable:

$f(a+h) - f(a) \approx hf'(a)$ (h close to 0)

Several Variables:

$$f(a+h, b+k) - f(a, b) \approx h\frac{\partial f}{\partial x}\bigg|_{(a,b)} + k\frac{\partial f}{\partial y}\bigg|_{(a,b)}$$

(h, k close to 0)

Derivatives of Common Functions

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\quad \frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$ $\quad \frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \frac{d}{dx}(\tan x) = \sec^2 x$

Exponential Differential Equation

If $y = f(x)$ satisfies $y' = ky$, then $y = Ce^{kx}$ for some constant C.

Useful Integration Facts

Indefinite Integral

$$\int f(x)\,dx = F(x) + C$$

provided that $F'(x) = f(x)$.

Fundamental Theorem of Calculus

Suppose that $f(x)$ is continuous on the interval $[a, b]$ with antiderivative $F(x)$; then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Riemann Sum Approximation

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0}[f(x_1) + f(x_2) + \cdots + f(x_n)]\Delta x,$$

where x_i is from the ith subinterval of $[a, b]$, of length Δx.

Midpoint Rule

$$\int_a^b f(x)\,dx \approx [f(x_1) + \cdots + f(x_n)]\Delta x,$$

where x_i is the midpoint of the ith subinterval.

Integration by Substitution

To determine

$$\int f(g(x))g'(x)\,dx:$$

1. Set $u = g(x)$, $du = g'(x)\,dx$.
2. Determine $\int f(u)\,du = F(u)$.
3. Substitute the value of u:

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$

Integration by Parts

$$\int f(x)g(x)\,dx = f(x)G(x) - \int f'(x)G(x)\,dx,$$

where $G(x)$ is an antiderivative of $g(x)$.

Integration Facts

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C, k \neq 0$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx [f(a_0) + 4f(x_1) + 2f(a_1) + 4f(x_2) + 2f(a_2) + \cdots + 2f(a_{n-1}) + 4f(x_n) + f(a_n)]\frac{\Delta x}{6},$$

where the a_i are the endpoints and the x_i are the midpoints of the subintervals.

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx [f(a_0) + 2f(a_1) + \cdots + 2f(a_{n-1}) + f(a_n)]\frac{\Delta x}{2},$$

where the a_i are the endpoints of the subintervals.